Prof. Kincade

March 21 — Review

Domain of composition of functions

$f(x) = \sqrt{x+2}$
$g(x) = \frac{3}{x-1}$ } Find the domain of $f(g(x))$

→ $f(g(x))$

Domain of 'g' ⇒ $\frac{3}{x-1}$ = ~ ⇒ $x \neq 1$

$x - 1 \neq 0$

$\boxed{x \neq 1}$

If $x = 1$, then $\frac{3}{x-1}$ is $\frac{3}{0}$, which is not allowed!

$f(\frac{3}{x-1}) = \sqrt{\frac{3}{x-1} + 2}$

Our domain for this must include the domain of both functions!

$\frac{3}{x-1} + 2 \geq 0$

$\frac{3}{x-1} \geq -2$

$3 \geq -2(x-1) \geq -2x+2$

$\boxed{-\frac{1}{2} \leq x}$

$f(x) = \sqrt{x+2}$

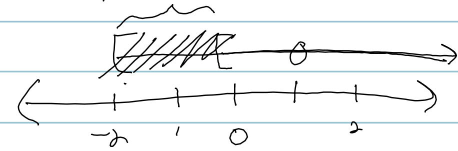
you may only take the square root of a positive number.

$x + 2 \geq 0$

$x \geq -2$

If x is a number more than -2, then the number under the radical will be positive!

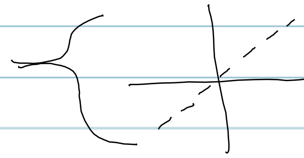
removed by $f(g(x))$



$[-\frac{1}{2}, 1) \cup (1, \infty)$

Inverse Functions

☆ one-to-one!



reflected about $y = x$

→ To prove that $f(x)$ and $g(x)$ are inverses...

$$f(g(x)) = x$$

AND

$$g(f(x)) = x$$

→ You can also <u>find</u> the inverse yourself!

1) let $y = f(x)$
2) Interchange the x's and y's.
3) Solve for y.
4) new $y = f^{-1}(x)$

☆ $f(x) = 5x - 3$

$y = 5x - 3$

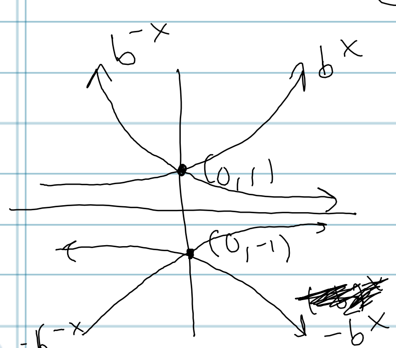
$x = 5y - 3$

$x + 3 = 5y \quad = x + 3$

$y = (x+3)/5) = \frac{x}{5} + \frac{3}{5} = \boxed{\frac{1}{5}x + \frac{3}{5}} = f^{-1}(x)$

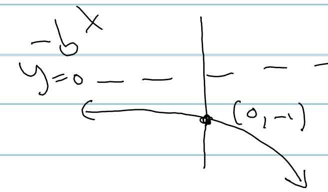
This is a necessity, NOT an option!

→ Graphing Logs & Exponents



$y = -2^{x-3} + 5$

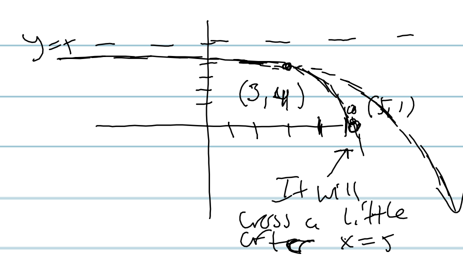
→ like $-b^x$



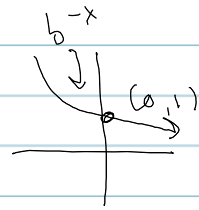
<u>Start</u> $(0, -1)$
<u>Shift</u> $(3, 5)$

$(3, 4)$

H.A. ⇒ $y = 5$

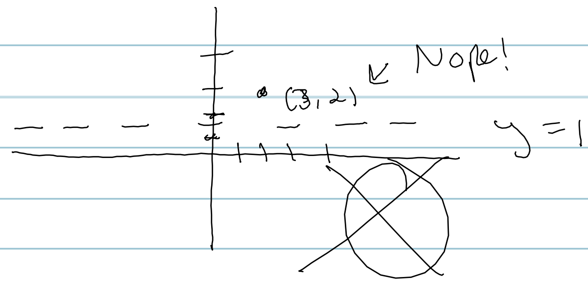


$y = \underbrace{2e^{3-x}}_{\text{like } b^{-x}} + 1 \Rightarrow$



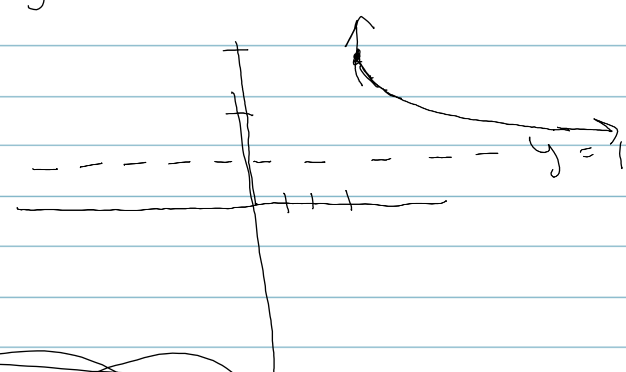
You didn't stretch!

Start: (0,1)
Shift: (3,1)
→ (3,2)
H.A. ⇒ y=1

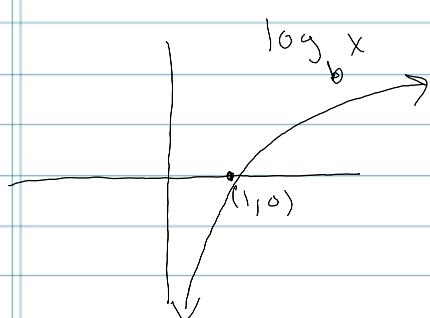


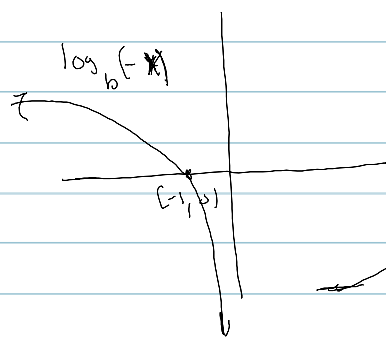
The trick is to remember you must include that stretch somewhere!

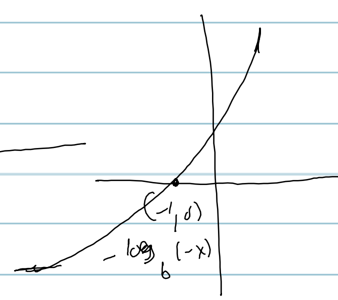
Start: (0, 2(1))
Shift: (3, 1)
(3, 3)
H.A. ⇒ y=1

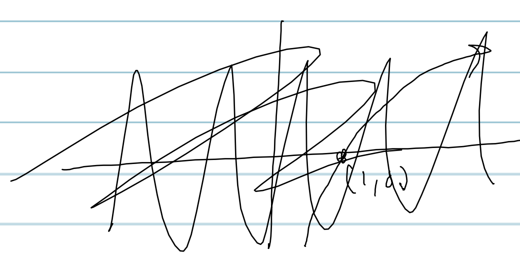


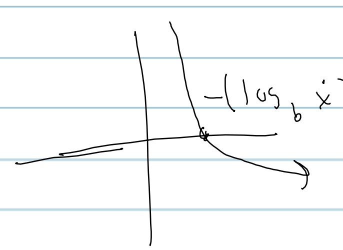
Logarithms







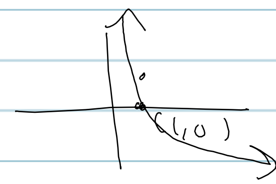




$y = -\ln(x-2) + 3$

$-\log_e(x-2) + 3$

LIKE: $-\log_b x$

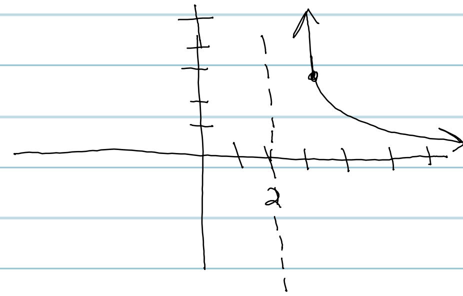


Start: $(1, 0)$

Shift $(2, 3)$

$(3, 3)$

V.A. $\ni$ $(x=2)$



Domain: $(2, \infty)$

Range: $(-\infty, \infty)$

$y = -2\log_3(4-\cancel{x}) + 1 \rightarrow$ like $-\log_b(-x)$

$\rightarrow 0!$

start: $(-1, 2(0))$

Shift: $(4, 1)$

$(3, 1)$

V.A. $\ni$ $x = 4$